UNIT- Ist

* STRESSES IN SOIL UNDER SURFACE LOADING *

*** INTRODUCTION →** Stresses are induced in a soil mass due to weight of overlying soil and due to the applied loads. These stresses are required for the stability analysis of the soil mass, the settlement analysis of foundations and the determination of the earth pressures. The stresses induced in soil due to applied load depend upon its stress-strain characteristics. The stress-strain behaviour of soils is extremely complex and it depends upon a large number of factors such as drainage conditions, water content, void ratio, rate of loading, the load level and the stress path.

*** Vertical Stresses due to a concentrated load →**

Boussinesq gave the theoritical solutions for the stress distribution in an elastic medium subjected to a concentrated load on its surface. The solution are commonly used to obtain the stresses in a soil mass due to externally applied loads. The following assumptions are made.

(1) The soil Mass is an elastic continuum, having a constant values of Modulus of elasticity (E) i.e, the ratio b/w the stress and strain is constant.

(2) The soil is homogeneous i.e, it has identical properties at different points.

(3) The soil is isotropic i.e, it has identical properties in all directions.

(4) The soil Mass is semi-infinite i.e, it extends to infinity in the downward direction and lateral directions. In other words, it is limited on its top by a horizontal plane and extends to infinity in all other directions.

(5) The soil is weightless and is free from residual stresses before the application of the load.

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where R = polar distance b/w the origin O and point P.

β = angle which the line OP makes with the vertical.

obviously, $R = \sqrt{x^2+y^2+z^2}$

or $R = \sqrt{r^2+z^2}$ where $r^2 = x^2+y^2$

and $\sin\beta = r/R$ and $\cos\beta = z/R$

The vertical stress (σ_z) at point P is given by.

$$\sigma_z = \sigma_R \cos^2\beta$$

$$\sigma_z = \frac{3}{2\pi}\left(\frac{Q\cos\beta}{R^2}\right)\cos^2\beta$$

$$\sigma_z = \frac{3Q}{2\pi}\frac{\cos^3\beta}{R^2}$$

$$\sigma_z = \frac{3Q}{2\pi}\frac{(z/R)^3}{R^2} = \frac{3Q}{2\pi}\cdot\frac{z^3}{R^5}$$

$$\sigma_z = \frac{3Q}{2\pi}\cdot\frac{1}{z^2}\cdot\frac{z^5}{R^5}$$

$$\sigma_z = \frac{3Q}{2\pi}\cdot\frac{1}{z^2}\left[\frac{z^5}{(r^2+z^2)^{5/2}}\right]$$

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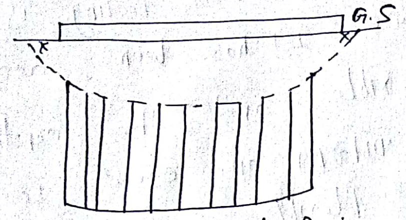
* **Contact pressure Distribution** → The upward pressure due to Soil on the underside of the footing is known as contact pressure. It has been assumed that the footing is flexible and the contact pressure distribution is uniform and equal to (q). Actual footings are not flexible as assumed. The actual distribution of the contact pressure depends upon a number of factor's Such as the elastic properties of the footings Material and Soil, the thickness of footing's.

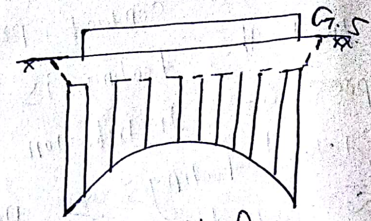
* **contact pressure on Saturated clay** →

contact pressure distribution under flexible and rigid footing's resting on a Saturated clay and subjected to a uniformly distributed load (q). When the footing is flexible it deforms into the shape of a bowl with the maximum deflection at the centre. The contact pressure distribution is uniform.
If the footing is rigid, the settlement is uniform. The contact pressure distribution is minimum at the centre and the maximum at the edges. The stresses at the edges in real soils cannot be infinite as theoritically determined for an elastic mass.

In real soils, beyond a certain limiting values of stress the plastic flow occurs and the pressure becomes finite.



(a) flexible footing

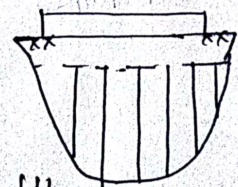


(b) Rigid footing

(fig - contact pressure on saturated clay)

* <u>Contact pressure</u> on <u>Sand</u> → In this case, the edges of the flexible footing undergo a larger settlement than at the centre. The soil at the centre is confined and, therefore, has a high modulus of elasticity and defects less for the same contact pressure. The contact pressure is uniform. If the footing is rigid, the settlement is uniform. The contact pressure increases from zero at the edges to a maximum at the centre. The soil being unconfined at edges, has low modulus of elasticity. However, if the footing is embedded, there would be ~~in~~ finite contact pressure at edges.

(a) flexible footing.



(b) Rigid footing

fig shows (contact pressure on Sand)

UNIT-II nd

Compressibility and consolidation

Introduction → when a soil mass is subjected to a compressive force, like all other materials, its volume decreases. The property of the soil due to which a decrease in volume occurs under compressive force is known as the compressibility of soil. The compression of soils can occur due to one or more of the following causes.

(1) compression of solid particles and water in the voids.

(2) compression and expulsion of air in the voids.

(3) Expulsion of water in the voids.

Expulsion → निष्कासन

* **Comparison of compaction and consolidation** →

1) consolidation is a gradual process of reduction of volume under sustained, static loading.
Whereas compaction is a rapid process of reduction of volume by mechanical means such as rolling, tamping and vibration.

(2) consolidation causes a reduction in volume of a saturated Soil due to squeezing out of water from the Soil.

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Whereas in compaction, the volume of a partially saturated Soil decreases because of expulsion of air from the voids at the unaltered water content.

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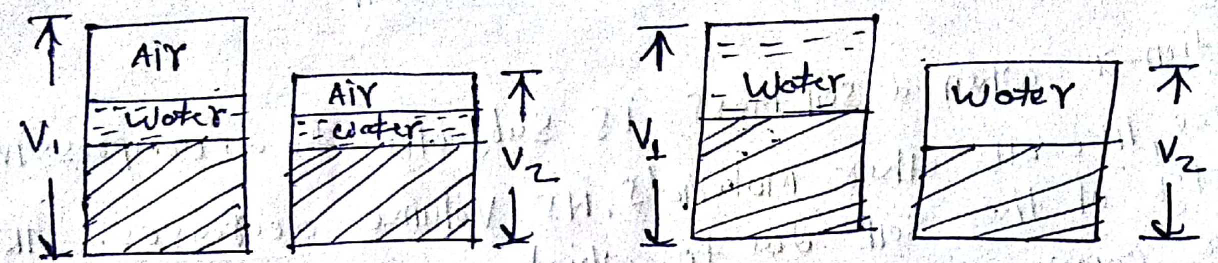


a (compaction)   (b) consolidation

(a) consolidation is a process which occurs in nature when the saturated soil deposits are subjected to static loads caused by the weight of the building and other structures.

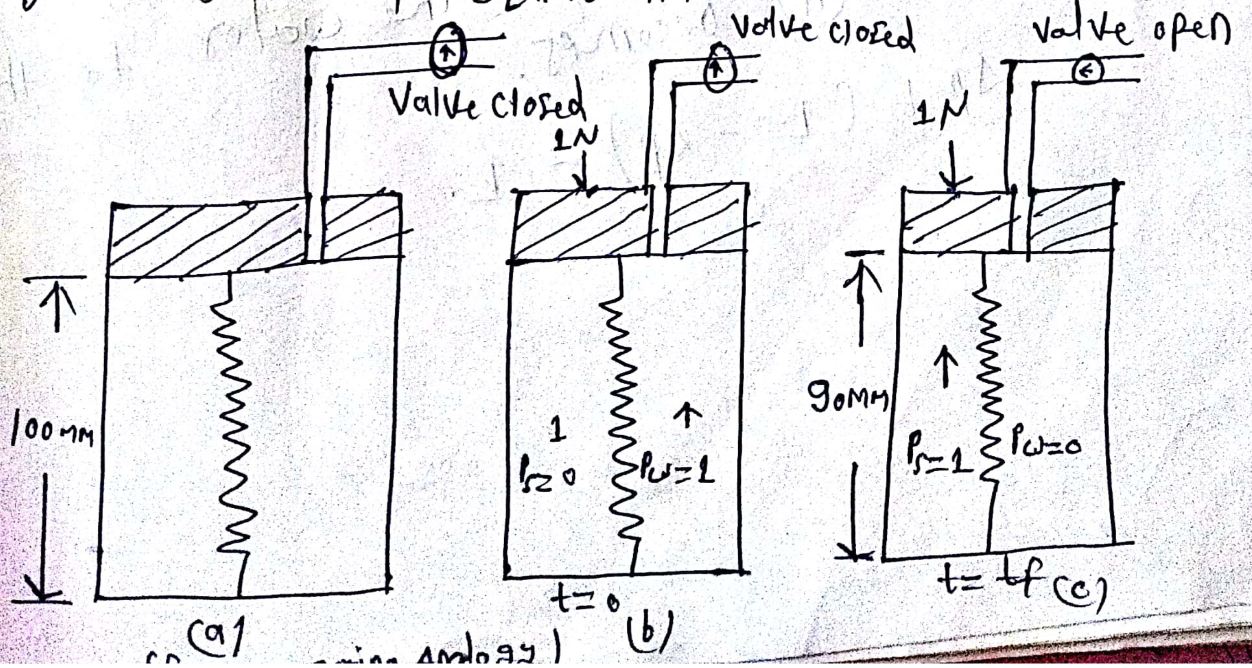
compaction is an artificial process which is done to increase the density (unit weight) of the soil to improve its properties before it is put to any use.

compaction of soil is required for the construction of earth dams, canal embankment, highways, runways and many other engineering applications.

* Spring Analogy For primary consolidation →

The process of primary consolidation can be explained with the help of the spring analogy given by Terzaghi fig shows a cylinder fitted with a tight fitting piston having a valve. The cylinder is filled with water and contains a spring of specified stiffness. Let the initial length of the spring be 100MM and the stiffness of spring be 10MM/N. Let us assume that the piston is weightless and the spring and water are initially free of stress.

When a load (P) (say 1 N) is applied to the piston, with its valve closed, the entire load is taken by water (fig shows b). The stiffness of the spring is negligible compared with that of water, and concequently no load is taken by spring from equilibrium.



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~~$P_w = P_s +$~~

$$\boxed{P_w + P_s = P} \longrightarrow (i)$$

where P_w = load taken by water
P_s = load taken by spring and P = total load.

For $P = 1\,N$ eq-(i) becomes.

$$P_w + P_s = 1$$

Initialy $(t=0)$ when valve is closed

$P_s = 0.00$ Therefore

$$\boxed{P_w = 1} \longrightarrow (ii)$$

If the valve is now gradually opened water starts escaping from the cylinder. The spring starts sharing some load and a decrease in its length occurs. When a portion (ΔP) of the load is transferred from the water to the spring eq-(ii) becomes.

$$\Delta P + (1.0 - \Delta P) = \underline{1}.$$

* Terzaghi theory of consolidation →

Terzaghi (1925) gave the theory for the determination of the rate of consolidation of a saturated soil mass, subjected to a static steady load.
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The theory is based on the following assumptions.

(i) The soil is homogeneous and isotropic.

(ii) The soil is fully saturated.

(iii) The solid particles and water in the voids are incompressible. The consolidation occurs due to expulsion of water from the voids.

(iv) The coefficient of permeability of the soil has the same value at all points, and it remains constant during the entire period of consolidation.

(v) Darcy's law is valid throughout the consolidation process.

(vi) Soil is laterally confined, and the consolidation takes place only in axial direction. Drainage of water also occurs only in the vertical direction.

(vii) The time lag in consolidation is due entirely to the low permeability of the soil.

(viii) There is a unique relationship b/w the void ratio and the effective stress, and this relationship remains constant during the load increment. In other words the ~~consolidation~~ coefficient of compressibility and the coefficient of volume change are constant.

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* Determination of coefficient of consolidation →

The curve b/w dial gauge Reading and time (t) obtained in the laboratory by testing the soil sample is similar in shape to the theoritical curve b/w (U) and (T_v) obtained from the consolidation theory. This similarity b/w the laboratory curve and the theoritical curve is used for the determination of the coefficient of consolidation (cv) of the soil. The methods are known as the fitting methods. The following two methods are commonly used.

(1) Square - root of time method →

(2) ~~Logai~~ Logarithm of time method →

(1) Square - root of time method → The devised by Taylor, utilizes the theoritical relationship b/w (U) and $\sqrt{T_v}$. The relationship is linear up to the value of (U) equal to about 60%. It has been further established that at (U = 90%), the value of $\sqrt{T_v}$ is 1.15 times the value obtained by the extension of the initial straight line portion (fig shows - a).

The sample of the soil whose coefficient of consolidation is required is tested as explained in Sect. 12.5. For a given load increment, the dial gauge reading are taken for different time intervals. A curve is plotted b/w the dial gauge Reading (R), as ordinate, and the $\sqrt{t}$ as absicssa (fig-b). The curve ABCDE shows the plot. The curve begins at the dial gauge reading R_0 at the time t_0 indicated by point A.

As the load increment is applied, there is an initial compression. It is obtained by producing back the initial linear path of the curve to intersect dial-gauge reading axis at point A'. This corresponds to the corrected zero reading (R_c). The consolidation b/w the dial gauge reading R_0 and R_c is the initial compression. The terzaghi theory of consolidation is not applicable in this range.

from the corrected zero reading point A', a line A'C is drawn such that its absissa is 1.15 times that of the initial linear portion A'B of the curve. The intersection of this line with the curve at point C indicates 90% of (U). The dial gauge reading corresponding to (C) is shown as R_{90} and the corresponding abiscissa as $\sqrt{t_{90}}$.

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$$\boxed{R_c - R_{100} = \frac{10}{9}(R_c - R_{90})}$$

The consolidation after 100% of primary consolidation in the range (DE) is the secondary consolidation the value of the coefficient of consolidation of the soil for that load increment is obtained from the value of $\sqrt{t_{90}}$ obtained from the plot. for $U = 90\%$ the value of $T_v = 0.848$. Therefore using Eq.

$$\boxed{c_v = \frac{T_v d^2}{t^2}} \doteq \frac{0.848 \times d^2}{(\sqrt{t_{90}})^2}$$

$$\boxed{c_v = \frac{0.848\, d^2}{t_{90}}} \quad \text{— (i)}$$

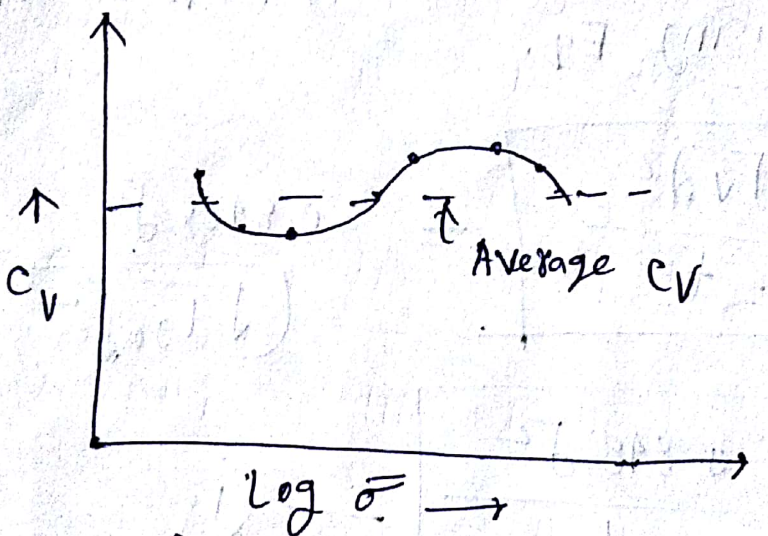
The distance of the drainage path (d) is half the total thickness. The total thickness may be taken as the average of the initial thickness (Hi) and final thickness (Hf) of the sample.

$$d = \frac{H}{2} = \frac{1}{2}\left[\frac{Hi + Hf}{2}\right) \quad \text{— (ii)}$$

For single drainage

$$d = H = \left(\frac{H_i + H_f}{2}\right) \quad \text{--- (iii)}$$

The test is repeated for different load increments and an average value of (Cv) obtained as shown in (fig - Variation of Cv).



(fig - Variation of Cv)

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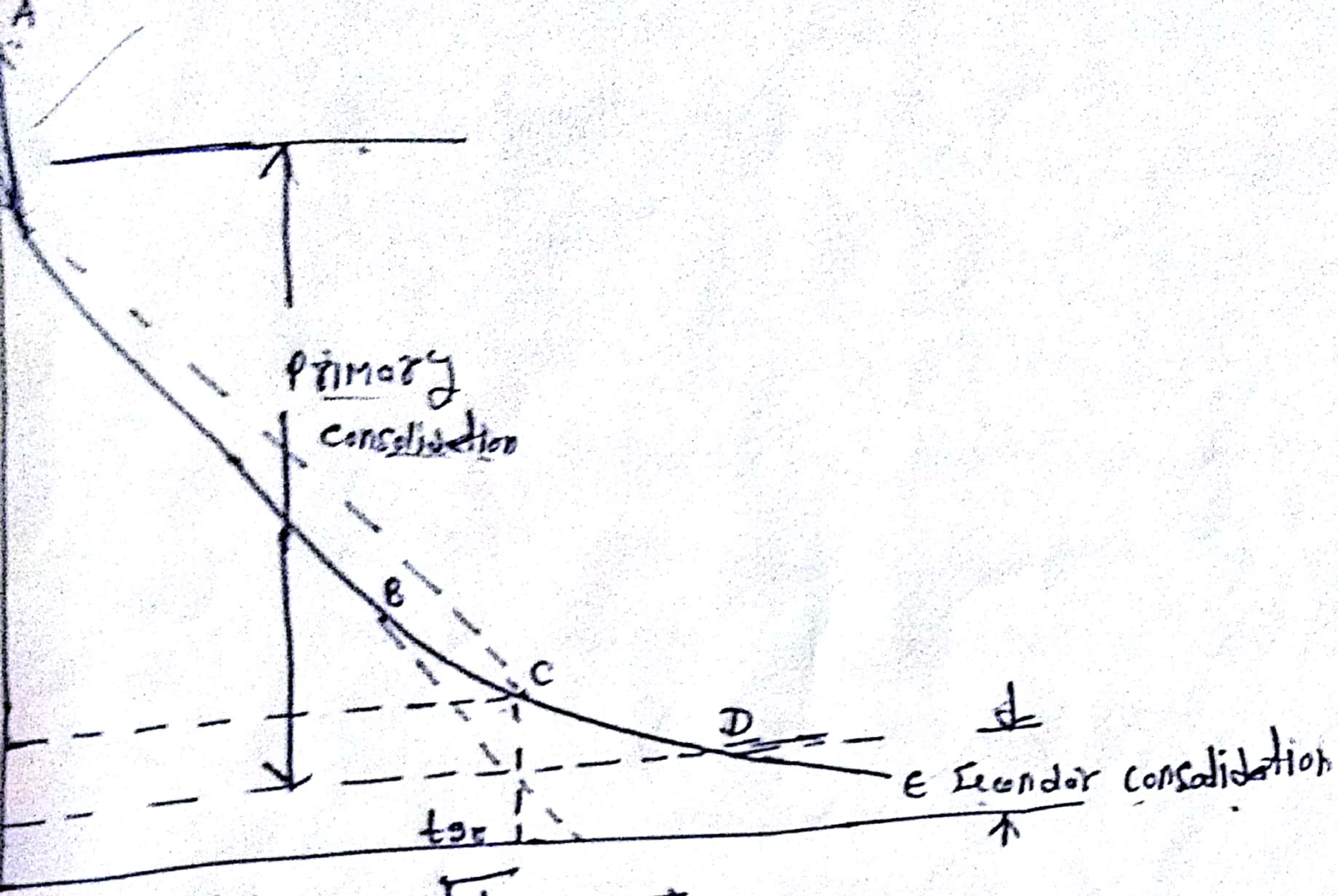
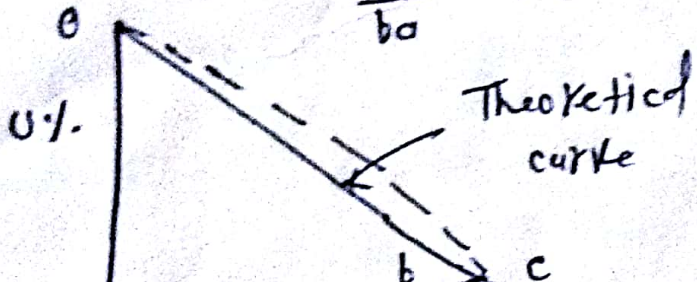


(b)

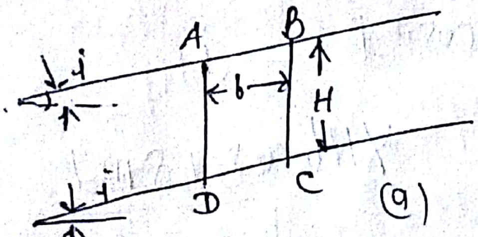
$$\frac{ca}{ba} = 1.15$$

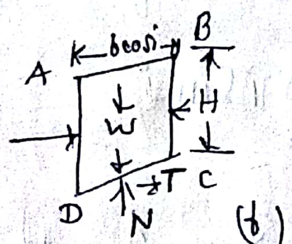


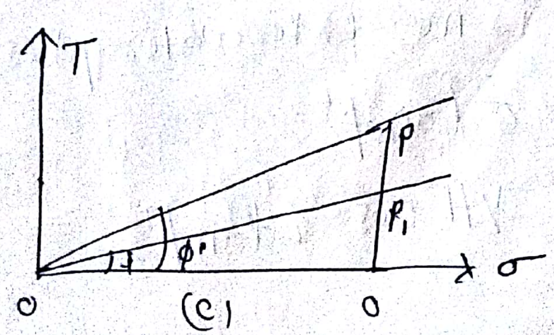
Stability of an infinite slope of cohesionless soils →

The stability criteria of an infinite slope of cohesionless soils will depend whether the soil is dry, or submerged or has steady seepage, as explained below.

1) **Dry Soil** → (fig-a) shows a section of an infinite slope having a slope angle of i. Let us consider the prism ABCD of the soil, with the inclined length AB equal to (b). The horizontal length of the prism is $b\cos i$. The height of the prism is H (fig-b).







(fig- shows infinite slope in dry sand).

Volume of prism per unit length = $Hb\cos i$

weight of prism per unit length

$$W = \gamma (Hb\cos i)$$

The weight of the prism can be resolved into the normal component N and tangential component T to plane CD.

Thus $N = W\cos i = \gamma Hb\cos^2 i$

$T = W\sin i = \gamma Hb\cos i\sin i$

The normal and shear stresses are given by.

$$\sigma = \frac{N}{b} = \frac{\gamma Hb\cos^2 i}{b}$$

$$\sigma = \gamma H\cos^2 i \quad \text{—(i)}$$

$$\tau = \frac{T}{b} = \gamma Hb\cos i\sin i$$

The factor of safety against shear failure is given by.

$$F_s = \frac{s}{\tau}$$

$$F_s = \frac{(\gamma H \cos^2 i)\tan\phi'}{\gamma H \cos i \sin i}$$

$$\boxed{F_s = \frac{\tan\phi'}{\tan i}}$$

(2) Submerged Slope → If the slope is submerged under water, the normal effective stress and the shear stress are calculated using the submerged unit weight and not the bulk unit weight as was used for dry soil.

$$\bar{\sigma} = \gamma' H \cos^2 i \quad \text{— (iii)}$$

$$\tau = \gamma' H \sin i \cos i \quad \text{— (iv)}$$

where γ' = submerged unit weight

Then factor of safety is given by.

$$F_s = \frac{s}{\tau} = \frac{(\gamma' H \cos^2 i)\tan\phi'}{\gamma' H \sin i \cos i}$$

$$F_s = \frac{\tan \phi'}{\tan i'}$$

it is observed that the factor of safety of a submerged slope is the same as that in dry condition.

(3) Steady seepage along the slope →

The forces acting on the vertical sides of the prism due to water and soil are equal and opposite and therefore cancel. The weight of the prism W is taken corresponding to the saturated conditions.

Therefore $\boxed{W = \gamma_{sat} H b \cos i}$

$$N = W \cos i = \gamma_{sat} H b \cos^2 i$$

$$T = W \sin i = \gamma_{sat} H b \sin i \cos i$$

As the base of the prism, there is an upward force due to water pressure (u) given by.

$$u = \gamma_w H \cos^2 i$$

uplift pressure

$$U = (\gamma_w H \cos^2 i) b$$

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Thus the net normal force ~~pressure~~ $\bar{N}$ is given by.

$\bar{N} = N - U$

$\bar{N} = \gamma_{sat} H b \cos^2 i - (\gamma_w H \cos^2 i) b$

or $\bar{N} = \gamma' b H \cos^2 i$

($\gamma' = \gamma_{sat} - \gamma_w$)

where γ' is submerged unit weight.

The effective stress is given by.

$$\bar{\sigma} = \frac{\bar{N}}{b} = \gamma' H \cos^2 i$$

Shear strength $s = \bar{\sigma} \tan \phi'$

$= \gamma' H \cos^2 i \tan \phi'$

Shear stress is given by.

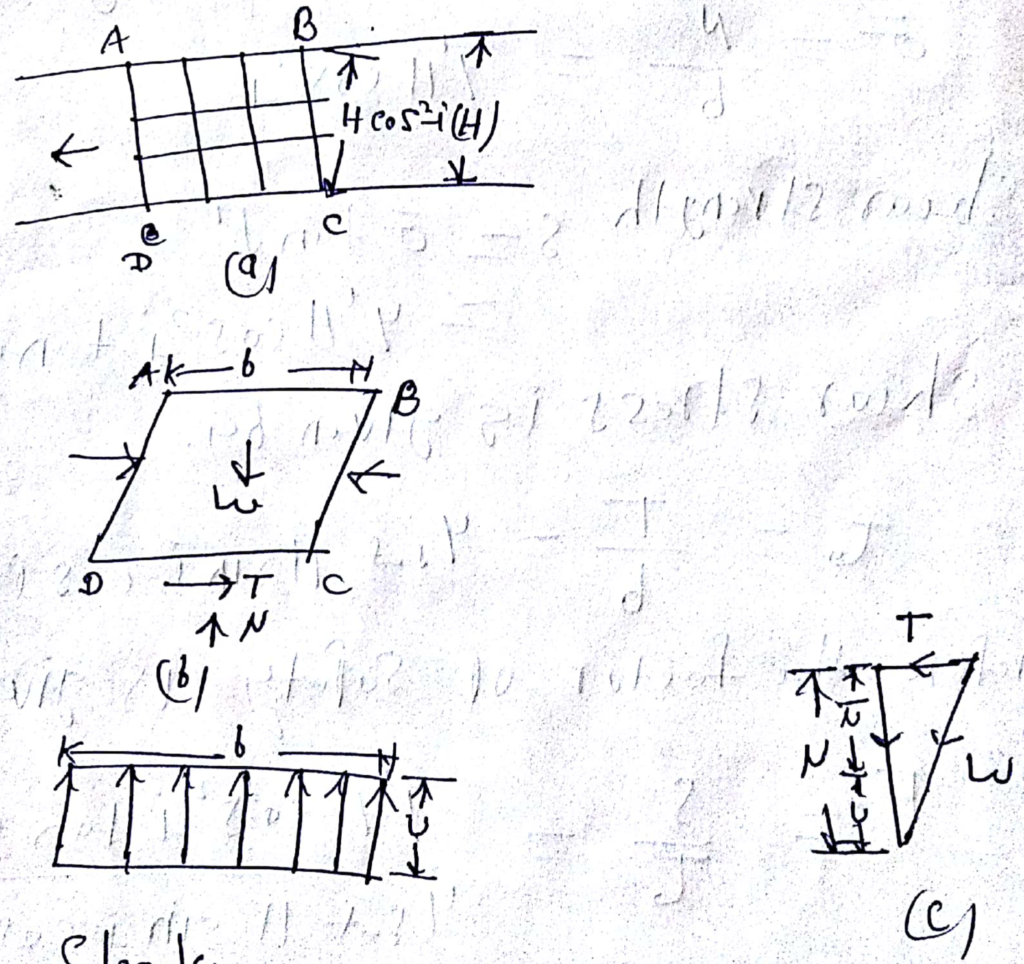
$$\tau = \frac{T}{b} = \gamma_{sat} H \sin i \cos i$$

Therefore the factor of safety is given by.

$$F_s = \frac{s}{\tau} = \frac{\gamma' H \cos^2 i \tan \phi'}{\gamma_{sat} H \sin i \cos i}$$

$$F_s = \frac{\gamma' \tan \phi'}{\gamma_{sat} \tan i}$$

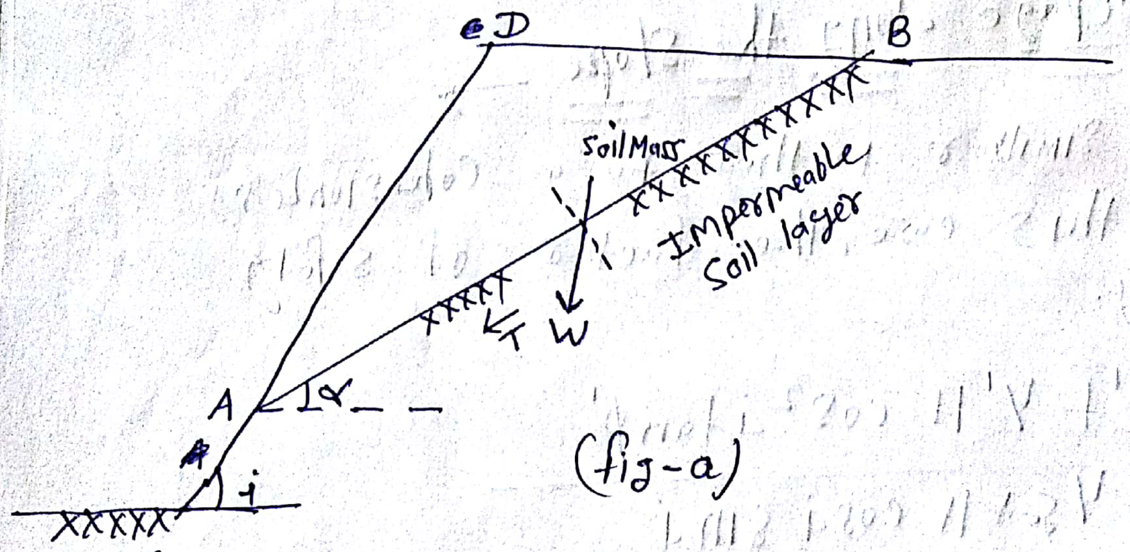
As the submerged unit weight γ' is about one-half of the saturated unit weight, the factor of safety of the slope is reduced to about one half of that corresponding to the condition when there is no seepage. The angle ϕ' in the wet condition of a cohesionless soil is approximately the same as in dry condition.



(fig Steady Seepage along the Slope.)

$$H_c = \frac{c'}{(\gamma_{sat} \tan i + \gamma' \tan\phi') \cos^2 i}$$

* **Wedge failure** → A wedge failure occurs when a soil deposit has a specific plane of weakness. The stratified deposits generally fail along the interface (fig-a). shows a Soil Mass resting on an inclined layer of impermeable soil. There is a tendency of the upper Mass to slide downward along the plane of contact AB.



(fig-a)

The force trying to cause sliding is the tangential component (T) of the weight (W) along the plane of contact.

$$\boxed{T = W\sin\alpha} \quad \text{---} \quad (i)$$

where α is the angle which the plane AB makes with horizontal. (W) is the weight of wedge per unit length perpendicular to the plane of paper.

The force tending to resist the sliding depends upon the cohesion (c) and the frictional force and ~~the~~ is given by.

$$\boxed{S = cL + (W\cos\alpha)\tan\phi} \quad \text{---} \quad (ii).$$

where L is the length of the failure surface AB.
The factor of safety against sliding is obtained from (i) and (ii) as.

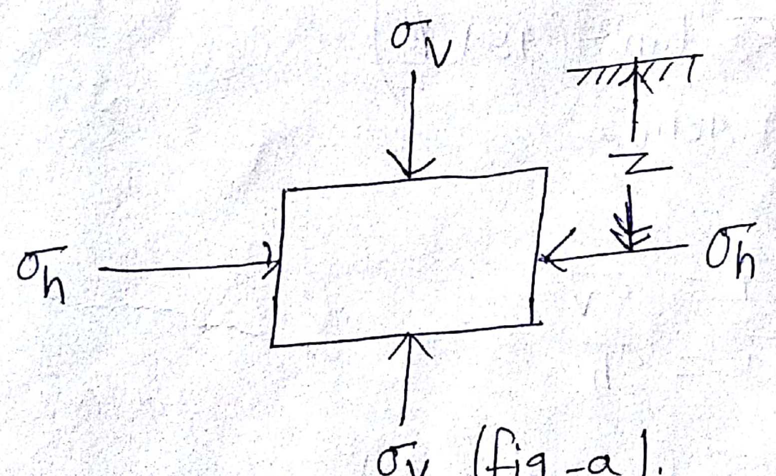
$$\boxed{F_S = \frac{cL + (W\cos\alpha)\tan\phi}{W\sin\alpha}} \quad \text{(iii)}$$

* Determination of earth pressure by Rankine's Theory for cohesionless Soil →

Following are the assumptions of the Rankine's theory →

1. The Soil mass is homogeneous and Semi-infinite.
2. The soil is dry and cohesionless.
3. The ground surface is plane, which may be horizontal or inclined.
4. The back of the retaining wall is smooth and vertical.
5. The soil element is in a state of plastic equilibrium i.e, at the verge of failure.

* Active earth pressure for cohesionless Soil →



(fig-a).

consider an element at a depth z below the ground surface. when the wall is at the point of moving away from the fill, the active state of plastic equilibrium is established, the horizontal pressure (σ_h) is then the minimum principal stress σ_3 and

The vertical pressure (σ_v) is the major principal stress σ_1 from plastic equilibrium equation.

$$\sigma_1 = \sigma_3 \tan^2\alpha + 2c\tan\alpha.$$

c for cohesionless soil

$$c = 0$$

$$\sigma_1 = \sigma_3 \tan^2\alpha$$

where $\alpha = 45 + \frac{\phi}{2}$

ϕ = Angle of internal friction.

$$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right)$$

In case of Active State.

$$\sigma_1 = \sigma_v$$

and $\sigma_3 = \sigma_h$

$$\sigma_v = \sigma_h \tan^2\left(45 + \frac{\phi}{2}\right)$$

$$\frac{\sigma_h}{\sigma_v} = \frac{1}{\tan^2\left(45 + \frac{\phi}{2}\right)}$$

$$\frac{\sigma_h}{\sigma_v} = \cot^2\left(45 + \frac{\phi}{2}\right)$$

But $\frac{\sigma_h}{\sigma_v} = K_a$, coefficient of active earth pressure.

$$K_a = \cot^2\left(45 + \frac{\phi}{2}\right)$$

σ_h = Lateral earth pressure = P_a

σ_v = Vertical pressure on the element $= \gamma z$.

$$\frac{P_a}{\sigma_v} = K_a$$

$$P_a = K_a \times \sigma_v$$

$\boxed{\therefore P_a = K_a \gamma Z}$ For cohesionless Soil.

$$K_a = \cot^2\left(45 + \frac{\phi}{2}\right).$$

$$= \frac{\cos^2\left(45 + \frac{\phi}{2}\right)}{\sin^2\left(45 + \frac{\phi}{2}\right)}$$

$\left(\because \begin{array}{l} 1 + \cos 2\phi = 2\cos^2\phi \\ 1 - \cos 2\phi = 2\sin^2\phi \end{array}\right)$

$$= \frac{\frac{1 + \cos 2\left(45 + \frac{\phi}{2}\right)}{2}}{\frac{1 - \cos 2\left(45 + \frac{\phi}{2}\right)}{2}}$$

$$K_a = \frac{1+\cos(90+\phi)}{1-\cos(90+\phi)}$$

And hence

$$\boxed{K_a = \frac{1-\sin\phi}{1+\sin\phi}}$$

→ This is the ~~active~~ Rankines ~~pressure~~ formula for active earth pressure.

* Terzaghi Bearing capacity theory ⟹ ***

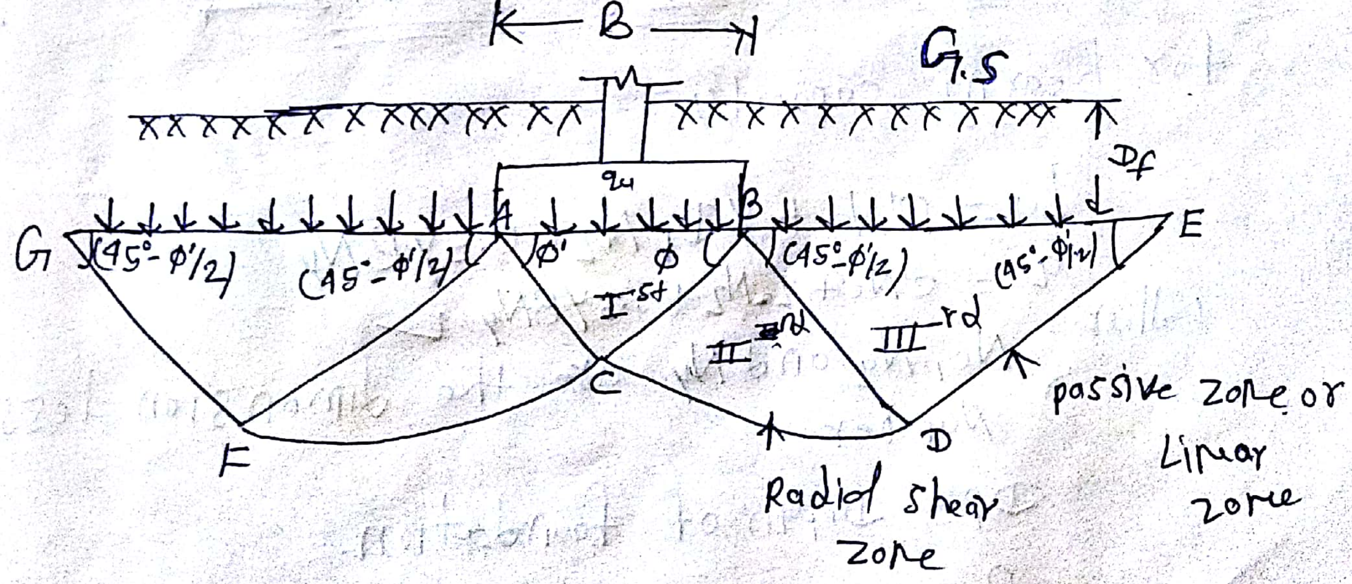
Terzaghi gave a general theory for the bearing capacity of soils under a strip footing, making following assumptions→

(1) The base of footing is rough.

(2) The footing is laid at a shallow depth i.e. $D_f \leq B$.

(3) The load on the footing is vertical and is uniformly distributed.

(4) The footing is long i.e. L/B ratio is infinite. Where B is the width and L is the length of footing.



(fig - Terzaghi Analysis)

⇒ As the base of the footing is rough, the soil in the wedge (ABc). The soil in this wedge (zone Ist) remains in a state of elastic equilibrium.
It is assumed that the angles CAB and CBA are equal to the angle of shearing resistance ϕ' of the soil. The sloping edges Ac and Bc of the soil wedge cBA bear against the radial shear zones CBD and CAF (zone IInd).
Two triangular zones BDE and AFG are the Rankine passive zones (zone-IIIrd).
An overburden pressure $\boxed{q = \gamma D_f}$ act as a surcharge on the Rankine passive zones.

* Terzaghi gave the following equations →
for bearing capacity

$$q_u = c'N_c + \gamma D_f N_q + 0.5\gamma B N_\gamma$$

$$q_u = c'N_c + q_o N_q + 0.5\gamma B N_\gamma$$

Where N_c, N_q and N_γ are the dimension less Number.

D_f = Depth of foundation.

B = width of foundation.

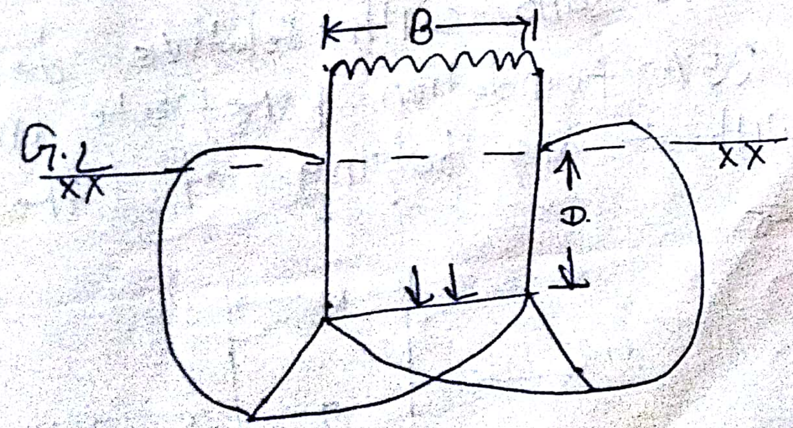
c' = cohesion

q = overburden pressure at the base of the footing.

γ = unit weight of soil at base level of foundation.

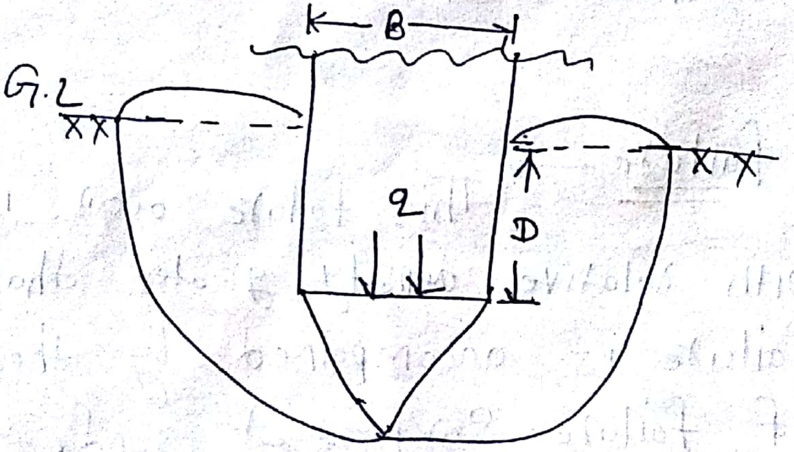
⇒ According to Terzaghi analysis bearing capacity equations depend upon nature of failure, because as failure is of three types
(i) General shear failure.
(ii) Local shear failure.
(iii) punching shear failure.

(i) General shear failure → This failure occurs in Sandy soil with relative density greater than 70%. The failure is accompained by the appearance of failure surface at sand. This failure is happened suddenly and this type of failure is designed as general shear failure by the Terzaghi.



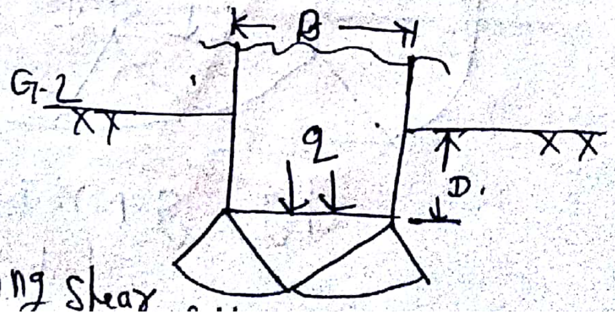
(fig - general shear failure)

(ii) Local Shear failure → when the relative density of sand lies b/w 35% to 70% then the failure is not sudden failure. This failure is accompained by appearance of failure surface at the sand surface with less bulging. This type of failure is known as Local Shear failure.



(fig - Local Shear failure)

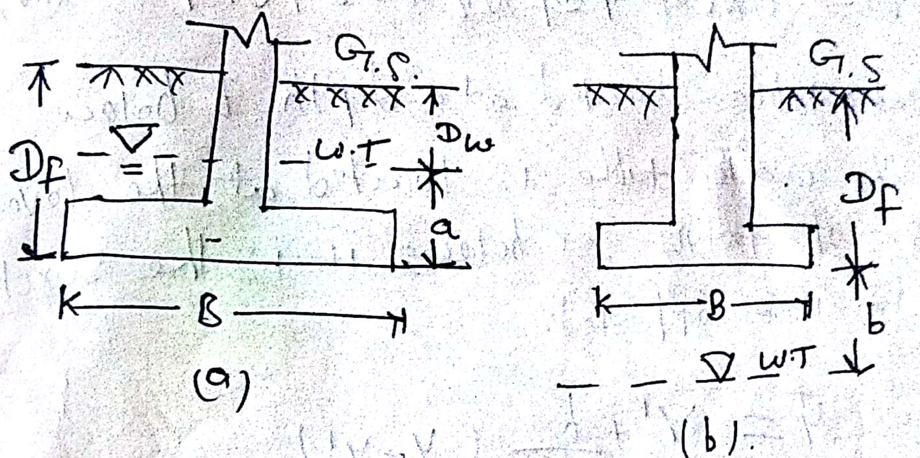
(iii) punching Shear failure → foundation or relatively loose sand with relative density less than 35% foundation peretrate in the soil without any bulging of the sand surface.



(fig - punching shear

* Effect of water table on bearing capacity →

for the ultimate bearing capacity has been developed based on the assumption that the water table is located at a great depth if the water table is located close to the foundation the bearing capacity equation needs,



(i) when water table located above the base of footing → The eff surcharge is reduced as the eff. weight below the water table is equal to the submerged unit.

$$q = D_w \gamma + a\gamma' \quad -(i)$$

D_w = depth of water table below the ground surface

a = height of water table above the base of footing.

Subsituting $a = D_f - D_w$

$$q = D_w \gamma + (D_f - D_w)\gamma'$$

$$q = D_w \gamma + D_f \gamma' - D_w \gamma'$$

$$D_f \gamma' + D_w(\gamma - \gamma') \quad -(ii)$$

$$q = cN_c + \gamma D_f N_q + 0.5\gamma B N_\gamma$$

And $$q_u = cN_c + q N_q + 0.5\gamma B N_\gamma \qquad \because q = \gamma D_f$$

$$q_u = cN_c + [D_f\gamma' + D_w(\gamma - \gamma')] N_q + 0.5\gamma B N_\gamma \quad \text{-(iii)}$$

If $D_w = 0$
i.e. $a = D_f$

$$q_u = cN_c + \gamma' D_f N_q + 0.5\gamma' B N_\gamma \quad \text{-(iv)}.$$

(ii) water table located at a depth b below base →
If the water table is located at the level of the base of footing or below it is the surcharge term is not affected.

$$\bar{\gamma} = \gamma' + \frac{b}{B}(\gamma - \gamma')$$

Where b = depth of water table below the base.
B = base width of the footing.

$$q_u = cN_c + \gamma D_f N_q + 0.5\gamma B N_\gamma$$

$$q_u = cN_c + \gamma D_f N_q + 0.5B\left[\gamma' + \frac{b}{B}(\gamma - \gamma')\right] N_\gamma$$

When $b = 0$ i.e W.T at the base.

$$\boxed{q_u = cN_c + \gamma D_f N_q + 0.5B\gamma' N_\gamma}$$

if $b = B$
i.e W.T at depth B below the base.

$$\boxed{q_u = cN_c + \gamma D_f N_q + 0.5B\gamma N_\gamma}$$

[illegible] when the ground water table is located at a depth equal to or greater than (B), there is no effect on the ultimate bearing capacity.